Bachelor’s Thesis

The Performance of Minimum Variance Portfolios in the Baltic Equity Markets

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Abstract

This paper applies minimum variance portfolio optimization to the Baltic equity markets and describes the out-of-sample performance of the optimized portfolios. The sample covariance matrix enhanced by Bayesian shrinkage procedure is employed to determine portfolio weights. The empirical results show that in the long run Baltic minimum variance portfolios have 20-30% lower volatility without the expense of lower returns compared to capitalization weighted market indices. Although such portfolios underperform the market indices in terms of returns during market upturns, they significantly outperform in a downtrend. Conclusions suggest that use of minimum variance portfolio optimization may substantially improve the investment performance of Baltic equity markets’ participants.

KEYWORDS: Markowitz, minimum variance portfolio, MPT, portfolio optimization
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List of Acronyms

CAPM – capital asset pricing model
EU – European Union
EWP – equally weighted portfolio
H1 – first half of the year
H2 – second half of the year
MPT – Modern Portfolio Theory
MVP – minimum variance portfolio
MVPs – minimum variance portfolios
MVPCAP – weight capped minimum variance portfolio
MVPU – minimum variance portfolio without upper weight limits
OMXBB – OMX Baltic Benchmark
OMXBBPI – OMX Baltic Benchmark price index
OMXBBCAPPI – OMX Baltic Benchmark weight capped price index
1 Introduction

By the beginning of 2009, capitalization weighted Baltic stock markets indices had lost over 70% of their value since the record highs of 2007. In addition, this dramatic decline had come with unseen volatility that was several times higher in the end of 2008 compared to historical figures. Such a turbulent situation greatly decreased investors’ appetite for risk and forced market participants to seek for defensive strategies that are less vulnerable to the market turmoil.

One of the investment strategies that are targeted at reducing portfolio risk is minimum variance portfolio (MVP) optimization, which is based on Markowitz’s (1952, 1959) Noble Prize winning Modern Portfolio Theory. The optimization procedure determines portfolio weights so that securities are diversified in the most efficient way and the portfolio has the lowest level of volatility. Haugen and Baker (1991), Clarke, Silva, and Thorley (2006), and Poullaouec (2008) stated that MVPs demonstrated returns similar to their benchmark capitalization weighted indices but with 25-30% lower standard deviation. Therefore, it comes as no surprise that minimum variance equity strategy has gained popularity and a number of investment funds and tradable indices that use MVP optimization have been launched since 2007 (Keefe, 2008; Appel, 2008).

However, academic research and practical applications of the promising MVP performance have been carried out only in the world’s largest financial markets. It is a question whether it would be possible to reduce portfolio volatility by a number at least close to 25% and not sacrifice returns in relatively small emerging markets, such as the Baltic States. Aiming to fill the gap in academic MVP research and provide insights for investors, I apply MVP optimization in the Baltic stock markets and test whether this investment strategy can bring tangible benefits to market participants holding funds in Estonian, Latvian, and Lithuanian securities.
I answer the research question “To what extent does the minimum variance portfolio optimization improve the investment results in the Baltic equity markets?” by testing the following hypotheses:

**Hypothesis I:** Minimum variance portfolios composed only of the stocks included in OMX Baltic Benchmark (OMXBB) index had lower volatility than their benchmarks in the years 2001–2008.

**Hypothesis II:** Minimum variance portfolios composed only of the stocks included in OMXBB index demonstrated returns not lower than their benchmarks in the years 2001–2008.

**Hypothesis III:** OMXBB was not mean-variance efficient in the years 2001-2008.

A method developed by Markowitz (1952) and applied by Clarke et al. (2006) serves as a basis for research. I form MVPs from the stocks included in OMXBB index and evaluate their out-of-sample performance against benchmark indices. A sample covariance matrix enhanced by Bayesian shrinkage procedure is employed to determine portfolio weights. I also account for the specifics of the Baltic markets, such as hardly ever practiced short sales, 10% weight limit for individual securities in an institutional investor’s portfolio, and semi-annually rebalanced indices.

The empirical results show that the investment strategy grounded on MVP optimization outperformed Baltic capitalization weighted market indices in the years 2001-2008. MVPs’ advantages were particularly noticeable in the recent period of a volatile market downturn.

The remainder of the paper is organized as follows. Section 2 provides a theoretical background and presents empirical studies of MVP. Section 3 describes data used for the analysis and defines methodology. Section 4 presents empirical findings that are later analyzed in Section 5. Section 6 concludes and overviews implications.
2 Review of Literature

Minimum variance portfolio optimization relies on Modern Portfolio Theory (MPT), which was introduced in 1950s by Harry Markowitz. The theory has stood the test of time and currently is one of the most important concepts in the financial portfolio selection (Santos, 2008). Its substantiability has been widely agreed upon by both academics and practitioners (Fabozzi, Gupta & Markowitz, 2002).

A brief introduction to MPT’s theoretical framework provided in this section should be sufficient for a reader to grasp the main ideas presented in the paper. More detailed descriptions may be found in the books written by Haugen (2001), and Elton and Gruber (2006), which I use as references in the following part.

2.1 Theoretical Framework of Modern Portfolio Theory

The main idea of MPT is to minimize risk (also referred as variance, standard deviation, and volatility) of an asset portfolio while not sacrificing expected returns (mean). The theory is based on the diversification effect, which forces the risk of a combination of assets to be lower than a simple average of the risk of individual assets. For instance, if there are two securities A and B with equal standard deviations of 10%, a portfolio holding both of these assets will have volatility lower than 10% unless A and B are perfectly correlated. The goal of risk minimization is achieved by changing weights of securities in a portfolio. Importantly, securities cannot be chosen according to individual characteristics only; their co-movement (correlation) with other securities has to be accounted for as lower correlation between assets strengthens the diversification effect. It is possible to derive a unique portfolio with the lowest risk for each level of expected returns. These unique portfolios are called mean-variance efficient as no other
A combination of available assets could have lower risk for a given level of expected returns. A set of all mean-variance efficient portfolios is called the efficient frontier, which is presented in Figure 1. A rational investor will only hold a portfolio that lies on the efficient frontier.

An important segment of MPT is minimum variance portfolio (MVP), which assigns weights to assets in such a way that portfolio risk is minimized. MVP has the lowest risk out of all mean-variance efficient portfolios and is independent of expected returns. Due to the diversification effect, MVP does not consist of a single stock with the lowest variance, but may contain even all stocks in an investment universe. This portfolio is positioned on the very left tip of the efficient frontier.

Fabozzi et al. (2002) claimed that the most often practiced application of MPT is asset allocation. Initially, investors have to determine assets they can invest in and any constrains they are subject to. The next step is to obtain estimates of returns, correlations, and volatilities of the investable securities. Then the estimates are used in an optimization procedure and an outcome that matches individual preferences is finally implemented in reality. Figure 2 depicts the MPT investment process.

![Figure 1. MPT (mean-variance) framework.](image)

*Note.* Composed by the author.
2.2 Criticisms and Alternatives

2.2.1 Estimation errors in expected returns

One of the weakest links in MPT investment process is the expected returns forecast. Traditionally, the Capital Asset Pricing Model (CAPM) was used as the main vehicle to predict expected returns. However, Roll (1977) and Fama and French (2004) concluded that CAPM was empirically not functional. As mean-variance optimization is extremely sensitive to expected returns, any errors in them might make the outcome far from optimal (Jorion, 1985; Best & Grauer, 1992). Jorion (1985) added that MPT’s sensitiveness to expected returns significantly downgraded its out-of-sample performance.

As a solution to the problem, factor models, such as Arbitrage Pricing Theory (APT), have been tested in place of CAPM (Yli-Olli & Virtanen, 1992). However, Jagannathan and Ma (2003) claimed that estimation errors in expected returns were so large that nothing was lost in ignoring the returns altogether. Not surprisingly, a number of empirical research papers and
recent practical applications of MPT have been focused on MVP, which is the only segment of MPT that does not require estimation of expected returns and in that way avoids the riskiest part of MPT investment process.

2.2.2 Estimation errors in covariance matrix

Another crucial input in mean-variance optimization is the covariance matrix. The easiest way to estimate it is to assume that the covariance matrix in the future will be the same as the sample covariance matrix. However, Bengston and Holst (2002), and Ledoit and Wolf (2004) pointed out that such estimation method was subject to errors caused by outliers and non-stationary parameters that tended to be different from period to period. Therefore, optimal portfolio’s weights might be perverted.

A widely applied approach to reduce estimation errors in a covariance matrix is Bayesian shrinkage procedure. Its aim is to pull the most extreme parameters toward universally constant values and in that way systematically enhance the out-of-sample performance (Jorion, 1986; Ledoit & Wolf, 2004). In addition, Jagannathan and Ma (2003) suggested using data of higher frequency to achieve higher precision in estimators.

2.2.3 Optimization procedure

It is a well-known phenomenon that mean-variance optimizers occasionally suggest investing a large part of a portfolio into one or several assets (Jorion, 1985; Michaud, 1998; Zhou, 2008). For instance, if there are two perfectly correlated securities in a market and both of them have identical variances but 20% and 20.5% expected returns respectively, then the optimizer will suggest investing all funds in the second security. This solution is obviously neither optimal, nor stable as even small changes in the inputs would greatly change the answer (Kaplan, 1998). In addition, Michaud (1989) found that mean-variance optimization procedure
magnifies the effect of estimation errors as it overweighs securities with large estimated returns, negative covariances, and small individual variances, while these parameters are very likely to have large estimation errors, particularly when short sales are allowed.

Imposition of maximum weight constraint and restriction of short sales diminish error maximization effect and prevent a portfolio from gaining extreme weights (Michaud, 1989; Jorion, 1992; Jagannathan & Ma, 2003). If returns, variance, and covariance estimators were without errors, these constraints would, in theory, have negative effect. However, it is almost impossible to avoid estimation errors, thus the constraints are frequently used in practice.

2.3 Arguments for Minimum Variance Portfolio

Although theoretical and empirical academic studies have researched various aspects of MPT, its real-life practical applications have been mostly focused on MVP. A number of companies\(^1\) are running investment funds that base their strategy solely on minimum variance optimization (Johnson, 2008; Keefe, 2008). Furthermore, this particular segment of MPT has drawn attention not only from asset management companies, but also from stock exchanges and index providers\(^2\). In contrast, there is not a single investment fund or index that is solely based on any other mean-variance efficient solution derived according to MPT principles.

There are three major reasons for MVP’s prevalence. First, Chopra and Ziemba (1993), Jorion (1986), Jagannathan and Ma (2003), and DeMiguel, Garlappi, and Uppal (2007) all

\(^1\) Unigestion, Acadian Asset Management, Union PanAgora, SEI Institutional, LGT Capital Management, State Street Global Advisors, Robeco, Invesco, AXA Rosenberg, and Vesco.

\(^2\) Deutsche Börse is running tradable indices that track the performance of MVPs in Germany, France, Switzerland, the U.S., and Japan, whereas MSCI Barra is the provider of MSCI Minimum Variance World index.
empirically proved that MVP performed better out-of-sample than any other mean-variance efficient portfolio. Second, MVP is least affected by the criticism of MPT as MVP optimization is independent from expected returns forecast, which is the major source of estimation errors. Third, an increasing risk aversion among market participants stimulates creation of financial products with managed volatility and MVP related investments serve this need well.

2.4 Empirical Studies of Minimum Variance Portfolio

A number of research papers tested empirical feasibility of MVP optimization. A list of these studies and their results may be found in Appendix 1. I review the ones that are most relevant to exploration of Baltic MVPs. However, to the best of my knowledge, there have been no MVP studies done in emerging markets.

Haugen and Baker (1991) were the pioneers of MVP research. They employed MVP optimization to test the efficiency of the most comprehensive capitalization weighted index in the US equity population – Wilshire 5000 – in the period from 1972 to 1989. The authors constructed MVPs at the beginning of each quarter and imposed constraints of 1.5% of portfolio invested in one stock and 15% - in one industry. In addition, short sales were restricted. Haugen and Baker used sample covariance matrix, which was computed over the trailing period of two years using monthly returns. The authors found that MVPs persistently demonstrated higher returns (22% higher) and lower volatility (21% lower) than Wilshire 5000. The major conclusion of the paper was that “matching the market is an inefficient investment strategy”. Haugen and Baker’s work served as a basis for a number of later MVP studies.

Fifteen years later, Clarke et al. (2006) researched the performance of MVPs in the US equity market in the years 1968-2005. They constructed MVPs out of 1000 largest US stocks,
rebalanced them every month, and compared the out-of-sample performance against S&P 500. The study was similar to Haugen and Baker’s (1991), however the time frame was expanded and more recent covariance structuring models were used.

Clarke et al. (2006) tested the performance of MVPs obtained using a number of covariance estimation methods and imposing different constraints. The authors referred to the covariance matrix enhanced by Bayesian shrinkage procedure and estimated using rolling 5 year monthly returns in a base case. A restriction of short sales and an upper limit of 3% for individual securities’ weights were also applied. In other cases, the researchers tested 1 year daily instead of 5 year monthly returns, relaxed short sales assumption, imposed market neutrality constraints so that MVPs had similar ex ante characteristics to those of the market, and used principal components procedure to improve the covariance matrix.

The main empirical finding was that MVPs, compared to S&P 500, achieved approximately 25% lower volatility without the expense of lower returns. The authors concluded that security variance and covariance were predictable and MVPs added value over market capitalization weighted benchmarks. This conclusion confirmed Haugen and Baker’s results.

In another recent research of MVP, Nielsen and Aylursubramanian (2008) analyzed the simulated performance of MSCI Minimum Variance World Index (MSCI MV World Index) for the period spanning from June 1995 to December 2007. The index was rebalanced semi-annually and had constraints on minimum and maximum weights for individual securities as well as for industries. The authors documented that the simulated MSCI MV World Index demonstrated approximately 30% lower volatility than the MSCI World Index over the given period and had a beta of 0.7. In addition, the Sharpe ratios for the two indices were 0.67 and 0.45 respectively.
Although theoretically MVP should lie on the very left tip of an efficient frontier, the authors showed that MVP exceeded the expectations in reality (see Figure 3).

*Figure 3. Nielsen and Aylursubramanian’s empirical findings.*

*Note. Composed by the author.*

Poullaouec (2008) also based his research on the performance of MSCI Minimum Variance World Index and discussed factors to consider when investing in MVP. The author claimed that rough expectations of MVP performance should be returns reduced by approximately 1% (in absolute terms) compared to capitalization weighted benchmark index, volatility reduced by 30%, and a correlation of 0.9. In addition, MVP maintained higher than benchmark’s Sharpe ratio and had lower downside risk, but at the same time demonstrated lower upside potential. These findings were consistent with Nielsen and Aylursubramanian’s (2008) results.

To sum up, empirical studies of MVP provide strongly positive results of such investment strategy. All the literature on the subject indicates tangible benefits from MVP optimization and documents only minor drawbacks. Considering the promising performance of MVP in the developed markets, it is a question whether MVP optimization would capture significant diversification benefits in emerging markets. With the background of MVP and its research introduced, I turn to the application of MVP optimization in the Baltic equity markets.
3 Methodology

The research by Clarke et al. (2006) serves as the main reference for the methodology. This particular paper has been chosen for several reasons: it is one of the most comprehensive studies of MVP, relies on recognized previous research, covers improved parameter estimation methods, has been published in credible academic press, and is written by the authors who have practical experience of implementing MVP in the financial markets. However, the model used by Clarke et al. (2006) cannot be applied in the Baltics without modifications. There are three important factors that influence specific characteristics of the model. Firstly, short sales are hardly available and almost not practiced in the Baltic stock markets. Secondly, institutional investors in the Baltic States are not allowed to invest more than 10% of their portfolio in an individual security. Finally, a relatively low number of available liquid stocks precondition that individual outliers may have a stronger impact on MVP in Baltic stock markets than in the developed ones.

Clarke et al. (2006) used original Markowitz’s MPT as the basis for research. In order to account for the specifics of the Baltic markets, I apply an adjusted version of MPT, which is formally specified in Appendix 2. The model is implemented in five steps as shown in Figure 4.

![Figure 4. Application of the model.](image)

Note. Composed by the author.

This process is very similar to the one used by Clarke et al. (2006), however there are two major differences. First, I use one year daily instead of five year monthly data for parameter estimation.
Jagannathan and Ma (2003) agreed that this adjustment gives more observations (250 instead of 60), assumes stationarity of a covariance matrix for a shorter period (one year instead of five), and consequently increases precision. Second, I add statistical tests to check whether differences in volatilities, returns, and Sharpe ratios are significant.

The five-step process is repeated every half a year starting January 2001. Employing SmartFolio3 software for optimization, I determine weights for the two hypothetical MVPs: one that has individual weights capped at 10% (this portfolio will be further referred as MVPCAP), and another that does not have a maximum weight limit (MVPU). In order to address error maximization problems, Bayesian shrinkage procedure, proposed by Ledoit and Wolf (2004), is used in both cases to shrink the sample covariance matrix toward the constant correlations covariance matrix. In addition, short sales are restricted. I do not account for transaction costs as their effect with semi-annual rebalancing would be inconspicuous. To add one more perspective, I track the performance of a naively diversified portfolio (EWP for equally weighted portfolio) that invests equal amounts in all securities included in OMXBB and is rebalanced semi-annually.

Next, I observe the out-of-sample performance of all the hypothetical portfolios and compare it to their benchmarks. The performance is evaluated by testing three hypotheses. As the fundamental concern of this paper is portfolio risk reduction, Hypothesis I states that MVPs composed only from the stocks included in OMXBB index had lower volatility than their benchmarks in the years 2001-2008. This hypothesis can be formulated as

\[ \sigma_{\text{MVPCAP}} < \sigma_{\text{OMXBBAPPI}} \]
\[ \sigma_{\text{MVPU}} < \sigma_{\text{OMXBBPI}} \]

I fail to reject Hypothesis I if standard deviation of both MVPs is statistically significantly lower than their benchmarks and reject it otherwise. To measure statistical significance of the result, an
F-test (see Appendix 3 for a formal description of the test) is used with a null hypothesis of both standard deviations being equal.

\[
\frac{\sigma_{MVP\text{CAP}}}{\sigma_{OMXBB\text{CAPPI}}} = 1 \quad \text{and} \quad \frac{\sigma_{MV\text{PU}}}{\sigma_{OMXBB\text{PI}}} = 1
\]

Although the primary aim is to find whether MVP optimization can reduce portfolio risk, it is important to know whether this potential improvement does not come with the expense of lower returns. Therefore, **Hypothesis II**, which states that MVPs had returns not lower than their benchmarks, is tested:

\[
\text{Returns}_{MVP\text{CAP}} >= \text{Returns}_{OMXBB\text{CAPPI}}
\]

\[
\text{Returns}_{MV\text{PU}} >= \text{Returns}_{OMXBB\text{PI}}
\]

Hypothesis II is not rejected if none of the two Baltic MVPs has statistically significantly lower returns than their benchmarks. The hypothesis is rejected otherwise. Statistical significance is evaluated by employing t-test (see Appendix 3) with a null hypothesis of both comparative returns having the same mean:

\[
\text{Returns}_{MVP\text{CAP}} = \text{Returns}_{OMXBB\text{CAPPI}}
\]

\[
\text{Returns}_{MV\text{PU}} = \text{Returns}_{OMXBB\text{PI}}
\]

**Hypothesis III** links the previous two hypotheses by stating that OMXBB was not mean-variance efficient in the years 2001-2008. Mean-variance efficiency is evaluated by Sharpe ratios (SR), thus Hypothesis III can be formulated as

\[
\text{SR}_{MVP\text{CAP}} = \frac{\text{Returns}_{MVP\text{CAP}}}{\text{Standard deviation}_{MVP\text{CAP}}} > \text{SR}_{OMXBB\text{CAPPI}} = \frac{\text{Returns}_{OMXBB\text{CAPPI}}}{\text{Standard deviation}_{OMXBB\text{CAPPI}}}
\]

and

\[
\text{SR}_{MV\text{PU}} = \frac{\text{Returns}_{MV\text{PU}}}{\text{Standard deviation}_{MV\text{PU}}} > \text{SR}_{OMXBB\text{PI}} = \frac{\text{Returns}_{OMXBB\text{PI}}}{\text{Standard deviation}_{OMXBB\text{PI}}}
\]
Statistical significance of the results is grounded on Jobson-Korkie test (see Appendix 3), which is designed for comparison of Sharpe ratios. The test typically assumes that a risk-free rate is zero. Such an assumption is acceptable as this paper is concerned only about the relative performance of the hypothetical portfolios to stock market benchmarks and does not examine optimal amounts of equities, bonds, or other asset classes an investor should hold.

Hypothesis III is failed to be rejected if both MVPs have statistically significantly higher Sharpe ratios than their benchmarks. However, Jobson and Korkie (1981) and Jorion (1985) noted that the test has a low power and a high degree of difference is needed for significant findings. As MVPs and the market indices are comprised of the same relatively small set of securities, their performance is expected to be similar and potentially not significantly different. Therefore, if the differences between Baltic MVPs’ and their benchmarks’ Sharpe ratios are not statistically significant, then economic significance is evaluated.

4 Data

4.1 Index selection

I have chosen a euro-denominated market capitalization weighted index OMX Baltic Benchmark (OMXBB) as the index for the analysis. There are several factors that have influenced this decision. First, OMXBB captures all three Baltic stock markets (Estonian, Latvian, and Lithuanian). As these markets are highly integrated, it is practical to use a common benchmark rather than refer to three separate indices. Second, OMXBB contains only the most liquid stocks, which reduces the effect of thin trading and is important for implementation of optimized solutions. Third, the index often serves as a benchmark for investment funds that are investing in the Baltic stock markets (OMX Baltic index descriptions). Finally, the index is
available as price index (OMXBBPI), and weight capped price index (OMXBBCAPPI). The former version does not have any constraints, whereas the latter has a 10% upper limit for individual securities. I refer to OMXBBCAPPI as the benchmark for institutional investors, who are obliged to follow weight limits for individual securities. Meanwhile, OMXBBPI better reflects unconstrained individual investor’s profile, thus I use it as a benchmark for this type of market participants.

4.2 Sample

The sample for the analysis consists of the stocks that were included in OMXBB in the years 2001-2008. The beginning of 2001 is the first date when at least one year daily data, used for parameter estimation, become available for securities listed on the Baltic stock exchanges. The sample time frame covers 16 half-year sub-periods and provides a total of 2057 daily observations. A number of stocks included in the index ranges from 27 to 35. However, in order to reduce estimation errors, I exclude securities that have over 10% required data missing. On average, there is one security that is left out of the optimization process in each sub-period. However, these stocks are included later on when a sufficient amount of data accumulates. As OMXBB is rebalanced semi-annually, I adjust a combination of securities used in optimization procedure accordingly. With the same securities included, same constraints imposed, and same rebalancing frequency, both MVPs have very similar characteristics to their comparative indices. As all the Baltic currencies (Estonian kroon, Latvian lat, and Lithuanian litas) have been pegged to the euro and neither devaluation nor revaluation of them occurred during the researched period, an impact from exchange rate fluctuations does not have any influence on the results. All the data are obtained from the official webpage for the Baltic stock exchanges and filtered for new share issues, splits, and suspended trading.
5 Empirical Findings

Empirical findings are presented in four subsections. The first one is devoted to the performance of MVPCAP, the second – to MVPU, the third – to EWP, and the remaining section describes the results for the three hypotheses.

5.1 Weight Capped Minimum Variance Portfolio

The out-of-sample performance of MVPCAP has been superior to its benchmark OMXBBCAPPI. MVPCAP had an annualized standard deviation of 11.25% and on average demonstrated 4.15% returns every year, while OMXBBCAPPI’s respective numbers were 14.1% and 2.78%. Comparing these numbers in relative terms, volatility of MVPCAP was 20.2% lower and yearly returns were 49% higher. Figure 5 illustrates the performance of MVPCAP and OMXBBCAPPI.

![Figure 5. The performance of MVPCAP and OMXBBCAPPI.](image)

Although MVPCAP’s total result has been better than its benchmark’s, a breakdown of the whole sample into half-year sub-periods shows that the performance has been different in
different times (see Appendix 4). MVPCAP had lower volatility than OMXBBCAPPI in 14 out of 16 sub-periods and the two occurrences of underperformance were in 2001H1 and 2002H2. The largest difference of 31.1% (24.77% against 35.94%) in favor of MVPCAP was recorded in the second half of 2008, when volatility in the markets peaked. Furthermore, the returns of both portfolios were very similar until the year 2005 when OMXBBCAPPI started outperforming MVPCAP in a market uptrend. This outperformance was persistent till the market peaked in 2007. However, MVPCAP has been dominating OMXBBCAPPI since then and demonstrated a superior performance during the current period of crisis.

In addition, MVPCAP had a beta of 0.65 measured against the benchmark. The beta ranged from 0.52 to 0.83 in the sixteen sub-periods. The correlation coefficient between the returns of MVPCAP and OMXBBCAPPI was 0.82 and fluctuated from 0.58 to 0.90. Appendix 5 provides a comparison of exact numbers in the sub-periods.

5.2 Minimum Variance Portfolio without Upper Weight Limits

MVPU achieved its main goal of risk reduction but this has come with the expense of lower returns. MVPU had 28.7% lower annualized standard deviation than OMXBBPI as respective numbers for these two assets were 11.4% and 15.98%. Yearly returns of MVPU were 4.11%, which was 13.3% lower than OMXBBPI’s 4.74% returns. It is remarkable that MVPU had lower volatility in all sub-periods except for the first half of 2001, which was the earliest period analyzed. Both MVPU and OMXBBPI had almost identical returns in the period from 2001 to the beginning of 2005. Then MVPU started underperforming the market index in the uptrend that lasted until the first half of 2007. Since then, the hypothetical portfolio has been declining in a slower pace than its benchmark and the ending values of both assets for the period
spanning from 2001 to 2008 have equalized. Figure 6 and Appendix 4 illustrate and compare the performance of MVPU and OMXBBPI.

![Volatility Graph](image)

**Figure 6.** The performance of MVPU and OMXBBPI.

Furthermore, as presented in Appendix 5, MVPU had a beta of 0.49, which ranged from as low as 0.23 to as high as 0.74 in the sixteen sub-periods. The correlation coefficient between returns of MVPU and OMXBBPI for the whole period was 0.68 and ranged between 0.4 and a high of 0.84.

### 5.3 Equally Weighted Portfolio

EWP and value weighted market indices typically moved in a very similar fashion. However, annualized standard deviation of EWP was 11.3% and 21.7% lower than of OMXBKCAPPI and OMXBBPI. In addition, the hypothetical portfolio earned 4.32% on yearly basis, which was in between of 2.78% and 4.74% demonstrated by the market indices. In addition, compared to Baltic MVPs, EWP was approximately 10% more volatile but had 5% higher returns (see Appendix 4). EWP’s beta for the whole analyzed period was 0.81, whereas
the correlation coefficient with OMXBBCAPPI was 0.91 (see Appendix 5). Figure 7 shows the performance of EWP compared to OMXBBCAPPI.

![Figure 7. The Performance of EWP and OMXBBCAPPI.](image)

### 5.4 Results for Hypotheses

#### 5.4.1 Hypothesis I

A comparison of standard deviations shows that both MVPs had statistically significantly lower volatility than their benchmarks in the analyzed period (see Appendix 4). The null hypotheses of standard deviations being equal are rejected at 1% significance level. Therefore, Hypothesis I is not rejected.

A break down of the analyzed period shows that MVPCAP had significantly lower standard deviation than OMXBBCAPPI in 9 out of 16 sub-periods, while the market index did not significantly outperform the hypothetical portfolio in any of the sub-periods in terms of risk level. In addition, 7 of those 9 statistically significantly different performances came in 8 half-
year periods in 2005-2008. The standard deviation of MVPU was statistically significantly lower than OMXBBPI's in 12 sub-periods.

5.4.2 Hypothesis II

MVPCAP had higher returns than OMXBBCAPPI; meanwhile, MVPU demonstrated lower returns than OMXBBPI (see Appendix 4). However, both differences in returns were insignificant. The null hypotheses of MVPCAP’s daily returns being equal to OMXBBCAPPI’s, and MVPU’s – to OMXBBPI’s cannot be rejected even at 10% significance level. Therefore, Hypothesis II is not rejected. In addition, differences in all 16 sub-periods were insignificant.

5.4.3 Hypothesis III

With the Jobson-Korkie test assumption of a risk-free rate being zero, MVPCAP had a Sharpe ratio of 0.26, MVPU – 0.25, while comparative indices OMXBBCAPPI and OMXBBPI – 0.16 and 0.23 respectively. Although both MVPs had Sharpe ratios higher than their benchmarks, the differences were not statistically significant. Therefore, as argued in the Methodology part, such results are inconclusive and economic significance needs to be assessed.

MVPU’s Sharpe ratio was 8.7% higher than its benchmark’s OMXBBPI. Meanwhile, MVPCAP’s Sharpe ratio was 62.5% higher than OMXBBCAPPI’s, which is a substantial economic difference for investors. Therefore, Hypothesis III not rejected.
6 Analysis of the Empirical Findings

6.1 Full Sample

The empirical results are consistent with the empirical findings of MVP studies in developed markets. There are several important observations regarding Baltic MVPs’ performance and compositions.

Firstly, the confirmation of Hypothesis I implies that Baltic MVPs were less volatile than the market indices. As MVPCAP had 20.2% and MVPU – 28.7% lower volatility than their comparative indices, it is clear that this investment strategy would have helped to decrease portfolio risk by a substantial amount. Compared to MVPCAP, MVPU demonstrated better results because its benchmark OMXBBPI had large weights in a small number of securities and, consequently, was much more volatile in the period 2001H2-2005H1 than OMXBBCAPPI, which did not allow more than 10% to be invested in one stock. The extent of reduction in volatility is similar to the evidence from developed markets, where the number ranges between 20% and 30%. In addition, as presented in Figures 8 and 9, MVPs’ standard deviations were persistently lower throughout the whole analyzed period. MVP optimization was particularly efficient in the periods of higher volatility, such as the first half of 2007 and the second half of 2008. On the other hand, MVP optimization did not provide substantial gains when the situation in the market was relatively calm. This finding is consistent with Clarke et al. (2006). As a general inference, I claim that Baltic MVPs typically have at least 20% lower standard deviation than their benchmarks if market’s volatility is higher than 12%. If market’s volatility is less than 12%, then MVPs still have risk reduction effect, however of lower magnitude.
Secondly, the confirmation of Hypothesis II means that Baltic MVPs had returns statistically significantly not lower than their benchmarks. An absolute difference of 1.4% between MVPCAP’s and OMXBBCAPPI’s returns and of 0.6% between MVPU’s and OMXBBPI’s is not significant in economic terms either. Although the main objective of MVP optimization is risk reduction, Baltic MVPs’ long run returns were similar to those of market indices. This finding matches the conclusions of Haugen and Baker (1991), Clarke et al. (2006), and Nielsen and Aylusubramanian (2008). It is important to mention that Baltic MVPs strongly
underperformed market indices in terms of returns during the market upturn from 2005H2 to 2007H1, but caught up when a downtrend in the market took over (see Figures 5 and 6). Such performance of MVP is in line with Poullaouec’s (2008) statement that MVP has a lower downside risk but at the same time a lower upside potential.

Thirdly, the above-mentioned two points regarding volatility and returns imply that Baltic indices were not mean-variance efficient as it was possible to earn same returns with lower risk. Appendix 6 depicts the mean-variance framework with the empirical results from the Baltic States. Lower standard deviations of MVPCAP and MVPU place the portfolios leftwards their benchmarks in the mean-variance framework, but only MVPCAP is above its comparative index. Therefore, MVPCAP is clearly mean-variance superior to OMXBBCAPPI, whereas it is impossible to compare MVPU with OMXBBPI. My careful and conservative conclusion is that Baltic MVPs were at least as efficient as the market indices in the period from 2001 to 2008.

Fourthly, MVPCAP and MVPU had low beta coefficients (see Appendix 5). MVPCAP’s beta was 0.65 and MVPU’s – 0.49. This means that both portfolios were highly insensitive to general market movements. The main cause for this development was that stocks with low betas were little correlated with other securities in the market; therefore, they offered significant diversification benefits and tended to get relatively high weights in MVP. This effect was partially magnified by the constraint of short sales. If the short sales constraint was removed, the beta coefficient would be higher as strongly correlated securities could be shorted and in that way the impact from low beta stocks would be reduced. However, low betas are not surprising as they are a typical feature of MVP (Poullaouec, 2008). A superior performance of a low beta portfolio matches the findings of Blitz and Vliet (2007) and Thomas and Shapiro (2008), who claim that low volatility stocks exhibit higher risk-adjusted returns than the market portfolio.
Fifthly, I compare the performance of MVPCAP and MVPU against each other and try to determine the effect of the upper weight limit. Theoretically, if variance and covariance estimators were without errors, the weight constraint imposed on MVP should increase the variance of a portfolio as any constraint limits choices. In contrast, if estimators contained errors, then such constraint should cause shrinkage effect and improve the out-of-sample portfolio performance in terms of volatility. As MVPCAP had almost identical standard deviation as MVPU, it can be concluded that the sample covariance matrix enhanced by Bayesian shrinkage procedure did not contain severe errors, thus maximum weight constraint did not improve the performance. This finding is consistent with Jagannathan and Ma (2003), who claimed that upper weight limits, in contrast to short sales restriction, have little effect on portfolio’s performance.

Finally, equally weighted portfolio (EWP) demonstrated performance superior to the market indices. Its returns were almost identical to market returns; however the level of volatility, as shown in Figure 10 was almost constantly lower. This finding shows that a naive diversification strategy is a more efficient investment approach than replication of capitalization market indices, which is in line with DeMiguel et al. (2007) conclusions.

![Figure 10. Rolling 6-Month Volatility of EWP and OMXBBCAPPI.](image-url)
Having reviewed the performance of Baltic MVPs and EWP in the whole analyzed period, I split the full sample into three separate time frames with distinguishable features.

### 6.2 2001H1–2004H2

The first time frame spans from 2001H1 to 2004H2 inclusive. During this period, Baltic stock markets were little integrated, activity in terms of transactions and turnover was several times smaller than in recent times, and none of the three Baltic States was a member of the European Union (EU). In this period, yearly returns of MVPCAP were 30.13% with 9.37% annualized standard deviation, MVPU – 29.37% with 9.57%, OMXBBCAPPI – 27.73% with 9.94%, and OMXBBPI – 30.68% with 13.52%. All four gauges were increasing at a similar pace and with similar volatility. Nonetheless, OMXBBPI had apparently higher volatility as this market index was concentrated on a few stocks with a large market capitalization and suffered from lack of diversification effect.

MVPs did not consistently provide substantial benefits in this period neither in terms of returns nor in terms of risk reduction. The main reason for this development was low liquidity in the market. Modest activity of market participants meant that sometimes a number of stocks included in OMXBB were not traded for days, weeks, or even months, which caused MVP optimization solutions to be less accurate and forced MVPs to be more similar to random portfolios rather than the ones with the lowest risk. For instance, EWP had both lower volatility and higher returns than any of MVPs (34.44% returns with 8.52% volatility). MVPU still had 29% lower standard deviation than its benchmark OMXBBPI, but this difference was mainly determined by OMXBBPI's poor diversification.
6.3 2005H1–2007H1

Since 2005, all three Baltic stock exchanges had started using the same standards and regulations. In addition, Estonia, Latvia and Lithuania were accepted to the EU. These two factors and rapidly growing Baltic economies substantially increased the activity in the stock markets and a number of deals made in 2005 was almost twice as big as in 2004.

Higher liquidity enabled MVPs to achieve their goal of minimizing risk and MVPCAP had 17.53% lower volatility (9.24% against 11.2%) than OMXBBCAPPI, whereas MVPU was 22.3% less risky than OMXBBPI (9.32% against 11.99%). However, a strong market uptrend highlighted that MVPs underperform capitalization weighted indices in terms of returns in a booming stock market phase. Yearly returns of MVPs were much lower than their benchmarks as MVPCAP earned 4.18%, MVPU – only 0.06%, while OMXBBCAPPI surged 25.23% and OMXBBPI – 27.8%. In comparison, EWP’s performance – 20.25% returns with 10.36% standard deviation – was in between of market indices and MVPs: EWP was riskier than MVPs but earned more, while at the same time it was safer than the market indices but underperformed them in terms of returns. Summing up, MVP optimization significantly reduced portfolio risk during stock market boom phase in 2005H1-2007H1, however MVPs could not match market returns.

6.4 2007H2–2008H2

The third and the latest time frame spans from July 2007 to the end of December 2008. This period can be associated with the current economic crisis and crashing financial markets. Yet trading activity in the Baltic stock markets was slightly higher compared to the previous analyzed period.
Extreme volatility and stock prices going down at an unseen pace was the environment in which MVPs perform best. MVPCAP had 16.87% standard deviation paired with negative yearly returns of 42.69%, whereas respective numbers for MVPU were 17.17% and minus 37.86%. The performance of market indices was substantially worse in all measures: OMXBBCAPPI had volatility of 24.6% (28.7% more than MVPCAP’s) and declined 58.39% per year, while OMXBBPI’s standard deviation was 23.66% (30.21% higher than MVPU’s) and returns – minus 58.37%. The superior performance of Baltic MVPs was similar to the results for the same period posted by numerous investment funds that employ MVP optimization (Johnson, 2008). The most important factor that determined such performance of MVPs was their bias towards low beta value stocks (such as telecoms) that typically less vulnerable to market swings than growth stocks. In addition, as MVPs perform better in more volatile markets, turbulence that was present in the analyzed period contributed to the improved results as well.

7 Conclusions and Implications

This paper expands the existing literature of MVP studies to emerging markets and shows that results are similar to the findings in the developed markets. I have argued that MVP optimization provided an investment approach that outperformed a capitalization weighted Baltic stock market index OMXBB in the period from January 2001 to December 2008 inclusive. A weight capped version of MVP, which is designed to reflect institutional investor’s profile, had 20.2% lower standard deviation and 49% higher yearly returns than its benchmark OMXBBCAPPI. Minimum variance portfolio without upper limits, which is more applicable to individual investors, had 28.7% lower volatility and 13.3% lower returns than the comparative index OMXBBPI.
The paper shows that it is possible to develop an investment strategy based on MVP optimization that outperforms broad Baltic stock markets’ indices in the long run. The major features of Baltic MVPs are that they have 20-30% lower standard deviation than value-weighted indices, offer returns statistically insignificantly different from benchmarks’ returns, have a beta of 0.5-0.6 and a lower downside risk with a moderated upside potential. In addition, Baltic MVPs perform best in volatile and downward trending markets.

MVP investment strategy may potentially be attractive to institutional investors, especially for pension funds, who seek for defensive strategies during market downturn. Investment funds that are measuring their performance against Baltic stock market indices may enhance their performance by investing in MVP composed from Baltic stocks. As MVP is relatively not time consuming to implement and manage, it may find its niche among private investors as well. Furthermore, a growing interest in index investing and practice from Western European countries may stimulate inception of new Baltic indices that would track MVP performance. Such index could either be tradable or serve as an underlying for an ETF.

MVP optimization may be easily combined with other investment strategies based on fundamental and/or technical analysis. For instance, stocks chosen as inputs for the optimization procedure could be selected according to their fundamental values and ratios. Similarly, technical analysis tools could be employed for better entry and exit timing. In addition, portfolio rebalancing frequency might be tailored to individual preferences.

Finally, the fact that MVPs in the Baltics have similar characteristics to the ones in the developed markets stimulates further research into other developing countries and regions. Awareness of the empirical MVP results may influence behavior of investors, who are tackling a relevant topic how to operate under current volatile and downward trending market conditions.
8 Works Cited


The Performance of Minimum Variance Portfolios


9 Appendices

9.1 Appendix 1

Table 1

Studies of Minimum Variance Portfolios

<table>
<thead>
<tr>
<th>Source</th>
<th>Market, Benchmark</th>
<th>Analyzed Period</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Haugen &amp; Baker (1991)</td>
<td>U.S., Wilshire 5000</td>
<td>1972-1989</td>
<td>21% lower standard deviation and 22% higher returns</td>
</tr>
<tr>
<td>Chan, Karceski, and Lakonishok (1999)</td>
<td>U.S., value weighted index of 500 random stocks</td>
<td>1973-1997</td>
<td>16.7% lower standard deviation and 8.6% higher returns</td>
</tr>
<tr>
<td>Jagannathan &amp; Ma (2003)</td>
<td>U.S., value weighted index of 500 random stocks</td>
<td>1968-1999</td>
<td>21% lower standard deviation and 6% higher returns</td>
</tr>
<tr>
<td>Clarke, Silva, and Thorley (2006)</td>
<td>U.S., Russell 1000</td>
<td>1968-2005</td>
<td>28% lower standard deviation and 7% higher returns</td>
</tr>
<tr>
<td>Poullaouec (2008)</td>
<td>World, MSCI World</td>
<td>1988-2008</td>
<td>23% lower standard deviation and 3.5% higher returns</td>
</tr>
</tbody>
</table>

Note. All portfolios have upper weight limits and a short sales restriction.
9.2 Appendix 2

MPT claims that if variance is minimized for a certain level of returns, then it gives a point on the efficient frontier. MVP has weights of investable assets optimized in a way that the lowest possible variance is achieved independently of expected returns. The problem to find MVP can be defined as follows:

\[
\text{Minimize } \sigma_p^2 = \sum_{i=1}^{n} W_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} W_i W_j \sigma_{ij}, i \neq j
\]

Where \( \sigma_p^2 \): portfolio variance; \( W_i, W_j \): proportions invested in securities i and j; \( \sigma_i^2 \): variance of security i; \( \sigma_{ij} \): covariance between securities i and j; \( n \): a number of securities in a portfolio.

Furthermore, a budget constraint is imposed. It requires the portfolio to be fully invested (no cash can be left over) and does not allow borrowing additional funds. This is expressed as

\[
\sum_{i=1}^{n} W_i = 1
\]

In order to restrict short sales, all individual weights have to be non-negative. Thus

\[
W_i \geq 0 \text{ and } W_j \geq 0
\]

Finally, if an upper limit for an individual weight is imposed, it is specified as

\[
W_i \leq U \text{ and } W_j \leq U
\]

where U is the upper limit expressed in decimal.

As the objective function involves such terms as \( W_i^2 \) and \( W_i W_j \), it is a quadratic programming problem. With all the parameters defined, the minimization problem is solved by changing weights of securities.
9.3 Appendix 3

9.3.1 F-test

F-test is a variance comparison test. It performs a test on the equality of standard deviations (variances). A version used in this paper checks whether Variable 1 and Variable 2 have the same standard deviation. In addition, a two-tailed variant is specified, which tests against the alternative that the standard deviations are not equal.

$H_0: \sigma_1 = \sigma_2$

$H_{alternative}: \sigma_1 \neq \sigma_2$

Test statistic: $F = \frac{s_1^2}{s_2^2}$

where $s_1^2$ and $s_2^2$ are sample variances. The more this ratio deviates from 1, the stronger the evidence for unequal variances.

9.3.2 T-test

T-test is a mean comparison test. A version used in this paper treats the data as unpaired (two sets of data come from different variables) and tests that Variable 1 has the same mean as Variable 2. In addition, it is specified that the unpaired data is not assumed to have equal variances.

$H_0: \mu_1 - \mu_2 = 0$

$H_{alternative}: \mu_1 - \mu_2 \neq 0$

Test statistic: $t = \frac{\mu_1 - \mu_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$
Degrees of freedom: \( t = \frac{\left( \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2}{\frac{s_1^2}{n_1 - 1} + \frac{s_2^2}{n_2 - 1}} \)

where \( \mu \) is sample mean, \( s \) - sample standard deviation, and \( n \) - number of observations. The test statistic has a Student’s distribution if the null hypothesis is true.

9.3.3 Jobson-Korkie test

Jobson-Korkie test is a statistical test for equality of Sharpe ratios. The initial version of the test proposed by Jobson and Korkie (1981) is adjusted according to Memmel (2003) suggestion. The risk free rate is assumed to be zero. The test has a null hypothesis of equal Sharpe ratios:

\[ H_0: SR_1 - SR_2 = 0 \]

For this to hold, test statistics have to follow normal distribution. The Z score is calculated as follows:

\[ Z_{Jobson-Korkie} = \frac{s_1 \mu_2 + s_2 \mu_3}{\sqrt{\theta}} \]

with

\[ \theta = \frac{1}{n} \left( 2s_1^2s_2^2 - 2s_1s_2 \text{cov}_{1,2} + \frac{1}{2} \mu_2^2s_1^2 + \frac{1}{2} \mu_1^2s_2^2 - \frac{\mu_1\mu_2}{s_1s_2} \text{cov}_{1,2}^2 \right) \]

where

- \( s \): standard deviation
- \( \mu \): mean returns of an asset
- \( \text{cov} \): covariance between assets.
### 9.4 Appendix 4

#### Table 2

**Breakdown of Portfolios’ Performance**

<table>
<thead>
<tr>
<th>Period</th>
<th>MVPCAP</th>
<th>MVPU</th>
<th>EWP</th>
<th>OMXBPI</th>
<th>OMXBBCAPPI</th>
<th>Measure</th>
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<td>Returns</td>
<td>Returns</td>
<td>Returns</td>
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* - Statistically significantly different from benchmark at 1% level

** - Statistically significantly different from benchmark at 5% level

*** - Statistically significantly different from benchmark at 10% level
### 9.5 Appendix 5

Table 3

**Betas and Correlation Coefficients**

<table>
<thead>
<tr>
<th>Period</th>
<th>MVPCAP</th>
<th>MVPU</th>
<th>EWP</th>
<th>OMXBBPI</th>
<th>OMXBBCAPPI</th>
<th>Measure</th>
</tr>
</thead>
<tbody>
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*Note.* OMXBBCAPPI serves as a benchmark for MVPCAP and EWP, whereas OMXBBPI – for MVPU.
9.6 Appendix 6

Figure 10. Mean-variance framework.